

Exerciții la capitolul 2

2.1. De obținut expresiile asimptotice pentru integralele de mai jos în aproximația primilor doi termeni din descompuneri.

$$2.1.1. \quad \int_0^x \sqrt{\tau^2 + 1} d\tau, \quad x \gg 1;$$

$$2.1.3. \quad \int_0^x \sqrt[n]{\tau^n + 1} d\tau, \quad x \gg 1;$$

$$2.1.2. \quad \int_0^x \sqrt{\tau^4 + 1} d\tau, \quad x \gg 1;$$

$$2.1.4. \quad \int_0^x \frac{\ln \tau}{\tau + 1} d\tau, \quad x \gg 1;$$

$$2.1.5. \quad \int_0^x \left(1 + \frac{1}{\tau}\right)^\tau d\tau, \quad x \gg 1;$$

$$2.1.15. \quad \int_0^\infty \frac{e^{-\gamma\tau} \cos \gamma\tau}{\sqrt{\tau}} d\tau, \quad \gamma \gg 1;$$

$$2.1.6. \quad \int_0^x \tau \ln \tau d\tau, \quad x \gg 1;$$

$$2.1.16. \quad \int_0^\infty \frac{\ln^2 \tau}{\tau + 1} e^{-\gamma\tau} d\tau, \quad \gamma \gg 1;$$

$$2.1.7. \quad \int_0^x \tau^2 \ln \tau d\tau, \quad x \gg 1;$$

$$2.1.17. \quad \int_0^{\frac{\pi}{4}} \operatorname{tg}^n x dx, \quad n \gg 1;$$

$$2.1.8. \quad \int_0^x \tau \ln^2 \tau d\tau, \quad x \gg 1;$$

$$2.1.18. \quad \int_0^\pi \cos^{2n} x dx, \quad n \gg 1;$$

$$2.1.9. \quad \int_0^x \frac{\ln \ln \tau}{\sqrt{\tau + 1}} d\tau, \quad x \gg 1;$$

$$2.1.19. \quad \int_0^\pi \sin^n x dx, \quad n \gg 1;$$

$$2.1.10. \quad \int_x^\infty \frac{\sin \tau}{\tau} d\tau, \quad x \gg 1;$$

$$2.1.20. \quad \int_0^\pi \sin^n x \cos^2 x dx, \quad n \gg 1;$$

$$2.1.11. \quad \int_2^x \frac{d\tau}{\ln \tau}, \quad x \gg 1;$$

$$2.1.21. \quad \int_0^1 \frac{dx}{(1 + x^2)^n}, \quad n \gg 1;$$

$$2.1.12. \quad \int_0^x \tau e^\tau d\tau, \quad x \gg 1;$$

$$2.1.22. \quad \int_0^\infty e^{-\frac{\gamma}{2}\left(x + \frac{1}{x}\right)} dx, \quad \gamma \gg 1;$$

$$2.1.13. \int_x^{\infty} \sqrt{\tau^2 + 1} e^{-\tau} d\tau, \quad x \gg 1;$$

$$2.1.23. \int_0^{\infty} e^{-\frac{\gamma}{2} \left(\frac{x+1}{x} \right)} (x-1)^2 dx, \quad \gamma \gg 1;$$

$$2.1.14. \int_0^{\infty} \frac{e^{-\gamma\tau}}{\tau+1} d\tau, \quad \gamma \gg 1;$$

$$2.1.24. \int_0^{\infty} e^{-\frac{\gamma}{2} \left(\frac{2x+1}{x^2} \right)} dx, \quad \gamma \gg 1;$$

$$2.1.25. \int_0^{\infty} \frac{e^{-\gamma\tau}}{\tau^2 + 1} d\tau, \quad \gamma \gg 1;$$

$$2.1.37. \int_{-1}^1 \frac{e^{i\gamma\tau}}{\sqrt{1-\tau^2}} d\tau, \quad \gamma \gg 1;$$

$$2.1.26. \int_0^{\infty} \frac{\sin \gamma\tau}{\sqrt{1+\tau^2}} d\tau, \quad \gamma \gg 1;$$

$$2.1.38. \int_1^{\infty} \frac{e^{i\gamma\tau^2}}{1+\tau^2} d\tau, \quad \gamma \gg 1;$$

$$2.1.27. \int_0^{\infty} \frac{e^{-\gamma\tau^2}}{\sqrt{\tau+1}} d\tau, \quad \gamma \gg 1;$$

$$2.1.39. \int_0^{\infty} \frac{\sin \gamma\tau}{\sqrt{\tau}} d\tau, \quad \gamma \gg 1;$$

$$2.1.28. \int_0^{\infty} e^{-\frac{\gamma}{2} \left(\frac{2x+1}{x^2} \right)} dx, \quad \gamma \gg 1;$$

$$2.1.40. \int_0^{\infty} \frac{e^{-\varepsilon\tau}}{\sqrt{\tau^2 + 1}} d\tau, \quad 0 < \varepsilon \ll 1;$$

$$2.1.29. \int_0^{\pi} e^{-\gamma \sin x} dx, \quad \gamma \gg 1;$$

$$2.1.41. \int_0^{\infty} \frac{\ln \tau e^{-\varepsilon\tau}}{\sqrt{\tau+1}} d\tau, \quad 0 < \varepsilon \ll 1;$$

$$2.1.30. \int_0^{\pi} e^{-\gamma \sin x} x^2 dx, \quad \gamma \gg 1;$$

$$2.1.42. \int_0^1 \frac{\ln \tau}{\sqrt{\tau^2 + \varepsilon^2}} d\tau, \quad 0 < \varepsilon \ll 1;$$

$$2.1.31. \int_0^{\pi} e^{-\gamma \sin x} \cos^2 x dx, \quad \gamma \gg 1;$$

$$2.1.43. \int_0^1 \frac{d\tau}{\sqrt[4]{\tau^4 + \varepsilon^4}}, \quad 0 < \varepsilon \ll 1;$$

$$2.1.32. \int_0^{\infty} \frac{x^n e^{-\sqrt{n}x}}{(1+x^2)^n} dx, \quad n \gg 1;$$

$$2.1.44. \int_0^{\frac{\pi}{2}} \frac{d\varphi}{\sqrt{1-k^2 \sin^2 \varphi}}, \quad 0 < 1-k^2 \ll 1;$$

$$2.1.33. \int_0^{\infty} \frac{x^n n^{-x}}{(1+x^2)^n} dx, \quad n \gg 1;$$

$$2.1.45. \int_{-\pi}^{\pi} \frac{d\varphi}{\sqrt{1+k^2 - 2k \cos \varphi}}, \quad 0 < 1-k^2 \ll 1;$$

$$2.1.34. \int_0^{\infty} \frac{e^{-\gamma x^4 - x^2}}{x^2 + \gamma x^4 + 1} dx, \quad \gamma \gg 1;$$

$$2.1.46. \gamma^{\frac{3}{2}} \int_0^{\infty} \frac{(\sin \tau)^{\gamma}}{\tau^{\gamma-2}} d\tau, \quad \gamma \gg 1;$$

$$2.1.34. \quad \int_0^{\infty} \frac{\tau^{\alpha-1} e^{-\gamma\tau}}{\sqrt{\tau^2+1}} d\tau, \quad \gamma \gg 1;$$

$$2.1.47. \quad \int_1^{\infty} \frac{\tau e^{i\gamma\tau}}{\sqrt{\tau^2-1}} d\tau, \quad \gamma \gg 1;$$

$$2.1.35. \quad \int_0^{\infty} \tau^{\alpha-1} e^{-\gamma\tau} \ln \tau d\tau, \quad \gamma \gg 1;$$

$$2.1.48. \quad \int_{-1}^1 \frac{\tau^2 e^{i\gamma\tau}}{\sqrt{1-\tau^2}} d\tau, \quad \gamma \gg 1;$$

$$2.1.36. \quad \int_1^{\infty} \frac{e^{i\gamma\tau}}{\sqrt{\tau^2-1}} d\tau, \quad \gamma \gg 1;$$

$$2.1.49. \quad \int_1^{\infty} \frac{\tau e^{i\gamma\tau^2}}{1+\tau^2} d\tau, \quad \gamma \gg 1.$$